Method of separation of variables

Steps to solve :-

1. Assume the trial solution

$$u(x, y) = X(x) \cdot Y(y) \quad \text{or} \quad u = XY$$

Put the value of trial solution in given P.D.E.

2. Separate the variable and assume it equal to some constant K.

3. Take the separated variables individually and solve it. (find X and Y)

4. Put the value of X and Y in the trial ~~find~~ solution to given reqd. solution of given P.D.E.

Q using method of separation of variables

solve $\dfrac{\partial u}{\partial x} = \dfrac{2\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$

Soln. Given $\dfrac{\partial u}{\partial x} + \dfrac{2\partial u}{\partial t} + u$ — (1)

Let the solution of eqn (1) be

$$u = XT$$

$$u(x, t) = X(x) \cdot T(t)$$

Eq^n. (1) becomes

$$X'T = 2XT' + XT \Rightarrow (X' - X)T = 2XT'$$

$$\Rightarrow \frac{X' - X}{X} = \frac{2T'}{T} = K$$

Solving $\frac{X' - X}{X} = K \Rightarrow \frac{X'}{X} - 1 = K$

$$\frac{X'}{X} = K + 1$$

on integration $\log X \; (K+1)x + \log c_1$

$$= (K+1)x \cdot \log e + \log c_1$$

$$\Rightarrow \log X = \log e^{(K+1)x} + \log c_1$$

$$\log c_1 - e^{(K+1)x}$$

$$\Rightarrow X = c_1 e^{(K+1)x}$$

Solving $\frac{T'}{T} = \frac{K}{2}$

on integration

$$\log T = \frac{K}{2} t + \log c_2$$

$\Rightarrow \log T = \frac{k}{2} t \cdot \log e + \log c_2$

$\log e^{kt/2} + \log c_2$

$\log c_2 \cdot e^{kt/2}$

$\Rightarrow T = c_2 e^{kt/2}$

$\therefore u(x,t) = c_1 e^{(k+1)x} \cdot c_2 e^{kt/2}$

$\Rightarrow c_1 c_2 e^{(k+1)x + kt/2}$

At $t = 0$ $u(x,0) = c_1 c_2 e^{(k+1)x}$

Also $u(x,0) = 6e^{-3x}$ (given)

on comparing both eq^n.

$c_1 c_2 = 6, \quad k+1 = -3 \Rightarrow k = -4$

$\therefore$ The reqd. solution is

$u(x,t) = 6e^{(-4+1)x + (-4)t/2}$

$= 6e^{-3x - 2t}$

$\Rightarrow 6e^{-(3x+2t)}$ A