

# Method of separation of variables

Steps to solve :-

1. Assume the trial solution

$$u(x, y) = X(x) \cdot Y(y) \quad \text{or } u = XY$$

Put the value of trial solution in given PDE.

2. Separate the variable and assume it equal to some constant  $k$ .

3. Take the separated variables individually and solve it. (find  $X$  and  $Y$ )

4. Put the value of  $X$  and  $Y$  in the trial ~~find~~ solution to give reqd. solution of given PDE.

Q using method of separation of variables

solve  $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial t^2} + u$  where  $u(x, 0) = 6e^{-3x}$

sol<sup>n</sup>. Given  $\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial t^2} + u = 0$  — (1)

Let the solution of eq<sup>n</sup> (1) be

JULY 2010		AUGUST	
5	12 19 26 Mon	30	2 9 16 23
6	13 20 27 Tue	31	3 10 17 24
7	14 21 28 Wed		4 11 18 25
8	15 22 29 Thu		5 12 19 26
9	16 23 30 Fri		6 13 20 27
10	17 24 31 Sat		7 14 21 28



$$u(x, t) = X(x) \cdot T(t) \quad u = XT$$

Eq<sup>n</sup>. (1) becomes

$$X'T = gXT' + XT \Rightarrow (X' - X)T = gXT'$$

$$\Rightarrow \frac{X' - X}{X} = \frac{gT'}{T} = K$$

Solving  $\frac{X' - X}{X} = K \Rightarrow \frac{X'}{X} - 1 = K$

$$\frac{X'}{X} = K + 1$$

on integration  $\log X (K+1)x + \log C_1$

$$= (K+1)x \cdot \log e + \log C_1$$

$$\Rightarrow \log X = \log e^{(K+1)x} + \log C_1$$

$$\log C_1 = e^{(K+1)x}$$

$$\Rightarrow X = C_1 e^{(K+1)x}$$

Solving  $\frac{T'}{T} = \frac{K}{g}$

on integration

$$\log T = \frac{K}{g}t + \log C_2$$

2010		MAY					2010	
Mon	31	3	10	17	24	Mon	7	
Tue		4	11	18	25	Tue	8	
Wed		5	12	19	26	Wed	9	
Thu		6	13	20	27	Thu	10	
Fri		7	14	21	28	Fri	11	
Sat	1	8	15	22	29	Sat	12	
Sun	2	9	16	23	30	Sun	13	



$$\Rightarrow \log T = \frac{k}{2} t \cdot \log e + \log c_2$$

$$\log e^{kt/2} + \log c_2$$

$$\log c_2 \cdot e^{kt/2}$$

$$\Rightarrow T = c_2 e^{kt/2}$$

$$\therefore U(x, t) = C_1 e^{(k+1)x} \cdot c_2 e^{kt/2}$$

$$\Rightarrow C_1 c_2 e^{(k+1)x + kt/2}$$

$$\text{At } t=0 \quad U(x, 0) = C_1 c_2 e^{(k+1)x}$$

$$\text{Also } U(x, 0) = 6e^{-3x} \text{ (given)}$$

on comparing both eq<sup>n</sup>.

$$C_1 c_2 = 6, \quad k+1 = -3 \Rightarrow k = -4$$

$\therefore$  The reqd. solution is

$$U(x, t) = 6e^{(-4+1)x + (-4)t/2}$$

$$= 6e^{-3x - 2t}$$

$$\Rightarrow 6e^{-(3x + 2t)} \quad \text{A}$$

	JULY			AUGUST		
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	1	2	3	4